Karlsruhe Institute of Technology

Extending the Theory of Arrays: `memset`, `memcpy`, and Beyond

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```
struct list_node {
    int data;
    struct list_node *tail;
};
typedef struct list_node list;

list *reverse(list *l) {
  list *r = l, *p = NULL;
  while (r != NULL) {
    list *q = r;
    r = r->tail;
    q->tail = p;
    p = q;
  }
  return p;
}
```

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- SMT-solvers are routinely used in program analysis:
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  - Symbolic execution
  - Software bounded model checking
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  - ...
- Prominent theory: $\mathcal{T}_{\mathcal{A}}$ (theory of arrays)
  - Model arrays/structures/objects in the program
  - Model main memory

$\mathcal{T}_{\mathcal{A}}$: The Theory of Arrays

index terms	$t_{\mathsf{I}} ::= \ldots$
element terms	$t_{\mathsf{E}} ::= \ldots \mid \mathtt{read}(t_{\mathsf{A}}, t_{\mathsf{I}})$
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... and nothing else

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How to model standard library functions such as `memset` and `memcpy`?

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void *memset(void *dst, int c, size_t n);
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memcpy(a, b, 4);
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$$a' = \texttt{write}(a_3, 3, \texttt{read}(b, 3))$$

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Does not scale well for large constants

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$\Longrightarrow$ Extend $\mathcal{T}_{\mathcal{A}}$ by λ-terms that describe arrays

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Motivation

```
int i, j, n = ...;
int *a = malloc(2 * n * sizeof(int));
for (i = 0; i < n; ++i) {
    a[i] = i + 1;
}
for (j = n; j < 2 * n; ++j) {
    a[j] = 2 * j;
}
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$$a' = \lambda i.\, \mathrm{ITE}(0 \leq i < n,\ i + 1,\ \mathtt{read}(a, i))$$

$$a'' = \lambda j.\, \mathrm{ITE}(n \leq j < 2 * n,\ 2 * j,\ \mathtt{read}(a', j))$$

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β-reduction

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- Model memory mapped I/O
- Attaching metadata to memory regions (allocated, de-allocated, . . .)

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- Loops can often be automatically transformed into loops that satisfy these requirements

Example

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void filter_multiples(int p, int n)
{
    for (int j = p*p; j <= n; j += p) {
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Can be summarized using

$$a' = \lambda j.\ \mathrm{ITE}[p * p \leq j \leq n,\ \mathrm{ITE}((j - p * p) \bmod p = 0,\ 0,\ \mathtt{read}(a, j)),\ \mathtt{read}(a, j)]$$

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- Two quantifier-free approaches
  - Eager reduction
  - Instantiation-based approach

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- Does not provide a decision procedure in general

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- $\mathcal{T}_{\lambda\mathcal{A}}$ axioms are applied eagerly
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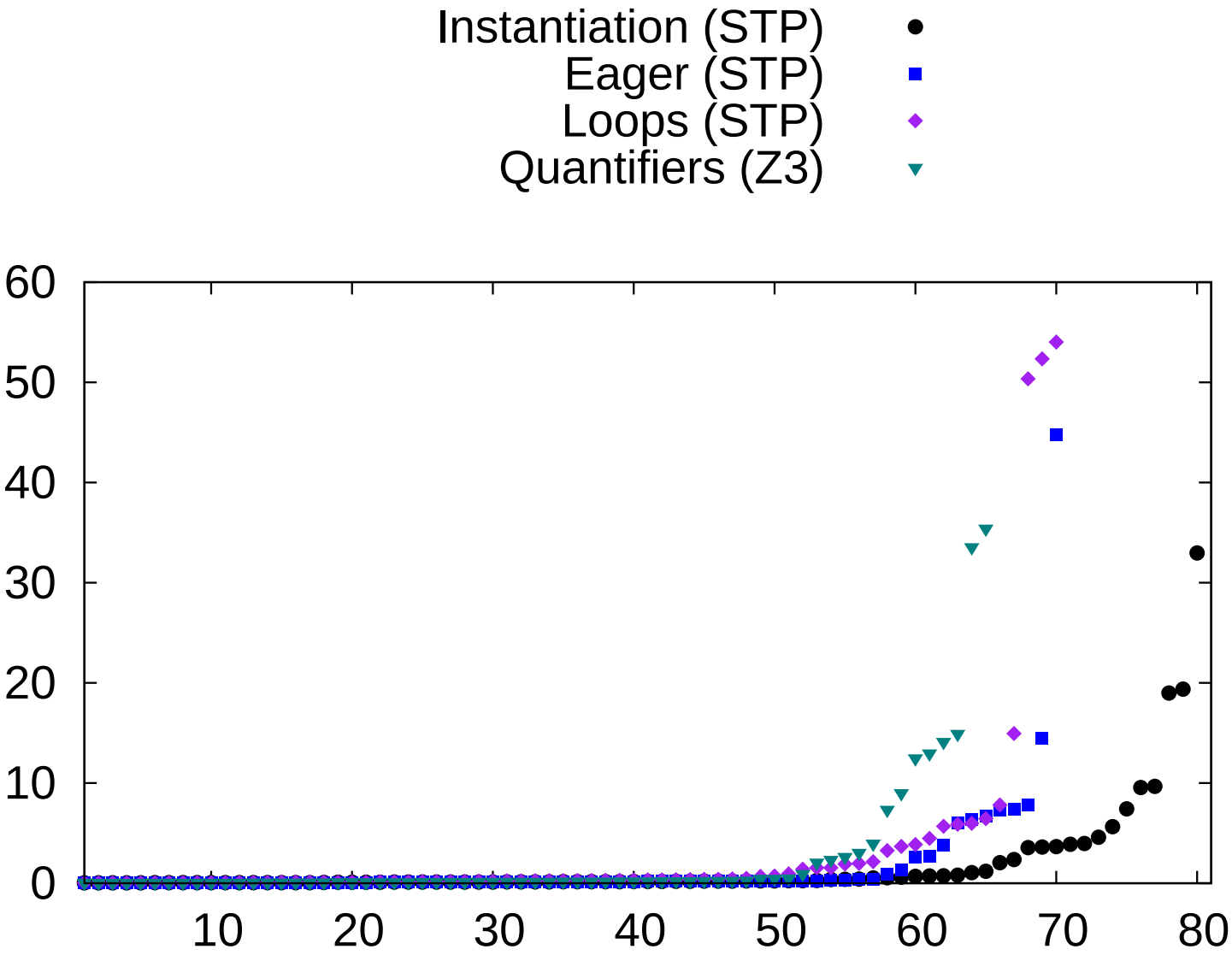
  - 14 program contain loops that can be summarized using λ-terms
- Of the resulting formulas, 20 are satisfiable and 61 are unsatisfiable
- Evaluated three reductions and loop unrolling
  - Quantifier-based approach using `Z3` and `CVC4`
  - Eager reduction and instantiation-based approach using `STP`, `Boolector`, `Z3`, and `CVC4`
  - Loop unrolling approach using `STP`, `Boolector`, `Z3`, and `CVC4`

Results

SMT solver	Approach	Total Time	# Solved Formulas	# Timeouts	# Aborts
STP	Instantiation	206.034	80	1	–
STP	Eager	779.544	70	11	–
STP	Loops	670.526	70	6	5
Boolector	Instantiation	818.782	71	10	–
Boolector	Eager	986.751	70	11	–
Boolector	Loops	1139.483	61	15	5
Z3	Instantiation	948.365	67	13	1
Z3	Eager	1043.632	66	15	–
Z3	Quantifiers	1122.489	65	16	–
Z3	Loops	1619.583	53	23	5
CVC4	Instantiation	928.079	67	14	–
CVC4	Eager	1119.748	65	16	–
CVC4	Quantifiers	1407.118	54	21	6
CVC4	Loops	1552.698	56	19	6

Results


Instantiation (STP)
Eager (STP)
Loops (STP)
Quantifiers (Z3)
60
50
40
30
20
10
0
10
20
30
40
50
60
70
80

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- Quantifier-free reductions perform better than `Z3`'s and `CVC4`'s reasoning involving quantifiers
- Integration into an SMT-solver using "Lemmas-on-demand"/"lazy instantiation" is the next step

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